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# Reformulation of Singh and Yadava Extended Model of Migration

## 1, Introduction

SINGH and Yadava (1978-79) have constructed an extended model of migration on the basis of the models which they had used earlier in the study of migration from a fixed Rural origin to a destination (1974). In the extended model they assumed that migration from a fixed place of origin $i$ to a definite place of destination $j$, $j = 1, 2, \ldots, n$ at time $t$, is a function of destination place opportunities at time $t$, prior migration at time $t - 1$, and intervening opportunities at time $t$ for the $j$th place of destination. Mathematically, the model can be specified as follows :

$$M_{jt} = f_1 (M_{jt-1}, O_{jt}, I_{jt}) \tag{1}$$

where

$M_{jt}$ .= No of migrants to the $j$th place of destination at time t

$M_{jt-1}$ = No. of migrants to the $j$th place of destination at time t — 1

$O_{jt}$ = number of opportunities at the place of destination at time $t$.

$I_{jt}$ — number of intervening opportunities connected with $j$th place of destination at time $t$.

They, then, made prior migration at time $t - 1$ (i.e. $M_{jt-1}$) as function of opportunities and intervening opportunities at time $t - 1$ and prior migration at time $t - 2$ (i.e. $M_{jt-2}$).

By assuming that prior migration to the place of destination begins from $t - m$ periods ago and prior migration at each time can be expressed as a function of opportunities and intervening opportunities at that time and migration prior to that time, they obtained the following functional relation :

$$M_{jt} = f_1 \left( f_2 f_3 \cdots f_{m-1} \left\{ f_m \left( M_{jt-m}, O_{jt-m+1}, I_{jt-m+1} \right) O_{jt-m+2} \right\} O_{jt}, I_{jt} \right). \tag{2}$$

Finally, they have derived a simplified expression of migration by assuming linear relationships among the functions $f_1, f_2, \cdots, f_m$, by taking the opportunities at the place of destination to be increasing exponentially and by treating the intervening opportunities as constant (see Singh and Yadava 1978-79).

The model, thus derived by Singh and Yadava, suffers from some limitations due to the non-inclusion of some factors affecting the phenomenon of migration. These are discussed below :

(i) Probably adopting the rationale of Nelson (1959) and Greenwood (1969, 1970) studies, Singh and Yadava have framed the mathematical relationship (1) by making the current migration ($M_{jt}$) at time $t$ as a function of prior migration at time $t$ — 1 (i.e. $M_{jt-1}$). But using $M_{jt-1}$ as an explanarory variable in model (1) will not serve properly the rationale of Nelson and Greenwood. This is because some from among the prior migrants at time $t$ — 1 (i.e. $M_{jt-1}$) would be returning or remigrating and some may die during the period ($t$ — 1, ()• Therefore, a portion of $M_{jt-1}$ will not be able to influence current flow of migrants in any way. Further, persons who migrated prior to time $t$ — 1 from the fixed origin $i$ to the destination $j$ would still be staying there at the time $t$ and will also influence current flow of migration. Hence, it is more appropriate that the model considers the stayers among all past migrants who came from the fixed origin $i$ to destination $j$, instead of taking the migrants at the time $t$ — 1 (i.e. $M_{jt-1}$).

(ii) Secondly, any model of migration should contain the characteristics of both the origin and the destination because migration from any fixed origin to any destination is the result of an imbalance in the distribution of opportunities or resources; and migration usually takes place from an area with low opportunities and facilities (origin) to an area having more opportunities and facilities (destination). Hence, migration can be said to be directly proportional to the opportunities at the place of destination and inversely proportional to the opportunities at the origin. Therefore, model (1) suffers due to non-inclusion of the characteristics of origin.

(iii) Lastly, Singh and Yadava called $I_{jt}$ as intervening opportunities, probably following Stouffer (1940). Instead, it might be better to call it by the broader name : "intervening factors" covering both barriers and opportunities.

## 2. Reformulation of the Model

The above limitations of the Singh and Yadava's Model (1) can be over-come by replacing the explanatory variable $M_{jt-1}$(i.e. prior migration at time $t$ — 1) by the stayers at time $t$ among past migrants, introducing $O_{it}$ (i.e. opportunities at time $t$) as origin characteristics and including $I_{jt}$ as intervening factors related to the destination $j$ and the origin $i$. With these modifications, the model can be reformulated as follows.

$$M_{jt} = K \frac{O_{jt}^{\beta_{1t}} S_{jt}^{\beta_{3t}}}{O_{it}^{\beta_{2t}} I_{jt}^{\beta_{4t}}} \xi_j \qquad (3)$$

where

$M_{jt}$ = no. of migrants to the destination $j$ at time $t$ from fixed origin $i, j = 1, 2, \ldots n$.

$O_{jt}$ = no. of opportunities at the place of destination $j$ at time $t$.

$O_{it}$ = No. of opportunities at the place of origin $i$ (fixed) at time $t$.

$S_{jt}$ = No. of stayers at time $t$ among the past migrants from fixed origin $i$ to destination $j$ at time $t$ — 1 and prior to time $t$ — 1.

$I_{jt}$ = No, of intervening factors between origin i (fixed) and destination $j$ at time $t$.

$\xi_j$ = random error; $K$ = proportionality constant.

As Singh and Yadava (1978-79) derived a simplified model by eliminating "prior migrants" (i.e. $M_{jt-1}$) using the variables opportunities and intervening opportunities; we may derive a simplified version of Model (3) by expressing the stayers at time $t$ as a function of opportunities and intervening factors,

Stayers among past migrants ($S_{jt}$) form a fraction of past migrants at or prior to time $t$ — 1. Suppose $MS_{jt-1}$ denote the number of past migrants (the migrant stock, as called by Greenwood), at time or prior to $t$ — 1, and $p$ is the proportion of past migrants ($MS_{jt-1}$) still staying upto time $t$ and (1 — $p$) proportion of past migrants returning or remigrating or dying during the period ($t$ — 1, $t$). Thus,

$$S_{jt} = P\, MS_{jt-1}. \qquad (4)$$

Then using relation (4), Model (3) can be written as

$$M_{jt} = K \frac{O_{jt}^{\beta_{1t}}}{O_{it}^{\beta_{2t}}} \frac{(p\, MS_{jt-1})^{\beta_{3t}}}{I_{jt}^{\beta_{4t}}} \xi_i. \qquad (5)$$

Now taking **logarithms,** equation (5) can be written as :

$$
\begin{aligned}
\log M_{jt} &= \log K + \beta_{1t} \log On — \beta_{2t} \log On \\
&\quad + \beta_{3t} \log p\ MS_{jt-1} — \beta_{4t} \log I_{jt} \\
&= (Po + Pa^* \log p)\ 4\ \beta_{1t} \log O_{jt} - pat \log O_{it} \\
&\quad + \beta_{3t} \log MS_{jt-1} - \beta_{4t} \log I_{jt}. \qquad (6)
\end{aligned}
$$

where

$\beta_0 = \log K\ 4\ \log \xi_j$

Now Greenwood (1969, 1970) has said, following Nelson's (1959) argument, that introducing the variable "past migrants" (migrants stock; $MS_{jt-1}$)as an explanatory variable in the model affect the estimated relationship to some extent, and has shown the same in his model using lag argument by expressing **the** migrant stock in terms of past effects of other variables. Similarly we can express log $MS_{jt-1}$in terms of past opportunities and intervening barriers and can express it in the form of lag argument by assuming that migration to the **destination** *j* began from past $t — m$ periods so that $MS_{ji-m-1} = O$ **as** follows :

$$
\begin{aligned}
\log MS_{jt-1} &= b_0 + b_{1t-1} (\log O_{it-1} + A \log O_{it-2} + \lambda^2 \log O_{it-3} \\
&\qquad 4 \ldots + \lambda^{m-1} \log O_{it-m}) \quad * \\
&\quad - b_{2t-1} (\log O_{it-1} + A \log O_{it-2} + \lambda^2 \log O_{it-3} \\
&\qquad + \ldots + \lambda^{m-1} \log O_{it-m}) \\
&\quad - b_{4t-1} (\log I_{it-1} + \lambda \log I_{it-2} + \lambda^2 \log I_{it-3} \\
&\qquad 4 \ldots + \lambda^{m-1} \log I_{it-m}) \qquad (7)
\end{aligned}
$$

**Following** Singh and Yadava (1978-79) we assume that the opportunities both at origin and destinations are increasing exponentially over the period at the rate of $r_i$ and $r_j$ respectively. Also we know that economic development is accompanied by increase in transportation and information flow facilities between origin and destinations which in turn decreases the force of the intervening barriers and makes the social transformation easy. **Hence,** we may assume that the intervening barriers decrease in force exponentially over the period of time at the rate of $h_j$ as economy develops. Mathematically

$$
\begin{aligned}
O_{jt-l} &= O_{jt-l+1}\, e^{-rj} \\
O_{it-l} &= O_{it-l+1}\, e^{-ri} \\
I_{jt-l} &= I_{jt-l+1}\, e^{hj} \qquad (8) \\
&l \text{ is fixed; and } j — 1, 2, \ldots n \\
&i \neq j;\ l = 1, 2, \ldots m
\end{aligned}
$$

Using the relations in (8), expression (7) can be written as.

$$\log MS_{jt-1} = b_0 + b_{1t-1} (\log On\, e^{-rj} + A \log On\, e^{-2rj} + A» \log Ojt\, e^{-3rj} + \ldots + \lambda^{m-1} \log O_{jt}\, e^{-mrj})$$

$$- b_{2t-1} (\log O_{it}\, e^{-ri} + A \log O_{it}\, e^{-2ri} + \lambda^2 \log O_{it}\, e^{-3ri} \; 4 \ldots 4 \; \lambda^{m-1} \log O_{it}\, e^{-mri})$$

$$- b_{4t-1} (\log I_{jt}\, e^{1hj} + A \log I_{jt}\, e^{2hj} + \lambda^2 \log I_{jt}\, e^{3hj} + \ldots + \lambda^{m-1} \log I_{jt}\, e^{mhj}) \quad (9)$$

$$= b_0 + b_{1t-1} (\log O_{jt} \; 4 \; A \log O_{jt} \; 4 \ldots + \lambda^{m-1} \log O_{jt})$$

$$- b_{1t-1} (\gamma_j + 2\gamma_j\lambda + 3\gamma_j\lambda^2 + \ldots + m\gamma_j\lambda^{m-1})$$

$$- b_{2t-1} (\log O_{it} + A \log O_{it} + A3 \log O_{it} + \ldots + \lambda^{m-1} \log O_{it})$$

$$+ b_{2t-1} (Yi + 2\gamma_i\lambda + 3\gamma_i\lambda^2 + \ldots + m\gamma_i\lambda^{m-1})$$

$$- b_{4t-1} (\log I_{jt} + A \log I_{jt} + \ldots + \lambda^{m-1} \log I_{jt})$$

$$- b_{4t-1} (h_j + 2h_j\lambda + 3h_j\lambda^2 + \ldots \; 4 \; mh_j\lambda^{m-1})$$

$$= b_0 + b_{1t-1} \left(\frac{1-\lambda^m}{1-\lambda}\right) \log O_{jt} - b_{1t-1} \left(\frac{1 + \lambda^m (m\lambda - m - 1)}{(1-\lambda)^2}\right) \gamma_j$$

$$- b_{2t-1} \left(\frac{1-\lambda^m}{1-\lambda}\right) \log O_{it} + b_{2t-1} \left(\frac{1 + \lambda^m (m\lambda - m - 1)}{(1-\lambda)^2}\right) \gamma_i$$

$$- b_{4t-1} \left(\frac{1-\lambda^m}{1-\lambda}\right) \log I_{jt} - b_{4t-1} \left(\frac{1 + \lambda^m (m\lambda - m - 1)}{(1-\lambda)^2}\right) h_j$$

$$\therefore \log MS_{jt-1} = b_0 + \left(\frac{1-\lambda^m}{1-\lambda}\right) (b_{1t-1} \log O_{jt} - b_{2t-1} \log On - b_{4t-1} \log I_{jt})$$

$$+ \left(\frac{1 + \lambda^m (m\lambda - m - 1)}{(1-\lambda)^2}\right) (-b_{1t-1}\gamma_j + b_{2t-1}\gamma_i - b_{4t-1}h_j). \quad (10)$$

Hence the variable migrant stock can be expressed in terms of present opportunities, and intervening barriers. By substituting (10) in (6) we have

$$\log M_{jt} = (\beta_0 \; 4 \; \beta_{3t} \log p + \beta_{3t} b_0) + \beta_{1t} \log O_{jt} - \beta_{2t} \log O_{it} - \beta_{4t} \log I_{jt}$$

$$+ \beta_{3t} \left[ \left(\frac{\cdot}{1-\lambda}\right) \left(b_{1t-1} \log O_{jt} - b_{2t-1} \log O_{it} \sim b_{4t-1} \log I_{jt}\right) \right.$$

$$\left. + \left(\frac{1 + \lambda^m (m\lambda - m - 1)}{(1-\lambda)^2}\right) \left(- b_{1t-1}\gamma_j + b_{2t-1}\gamma_i - b_{4t-1}h_j\right) \right] \quad (11)$$

Let $e^{\beta_0+\beta_{st}\log p+\beta_{3t}b_0} = \theta_0$ (constant)

$$\frac{1-\lambda^m}{1-\lambda} = \theta_1$$

and

$$\frac{1+\lambda^m(m\lambda-m-1)}{(1-\lambda)^2} = \theta_2$$

then expression (11) can be written as

$$M_{jt} = \theta_0 \frac{\left(O_{jt}^{\beta_{1t}+\beta_{3t}\theta_1 b_{1t-1}}\right) e^{\theta_2\beta_{3t}(-b_{1t-1}\gamma_j+b_{2t-1}\gamma_i-b_{4t-1}h_j)}}{\left(O_{it}^{\beta_{2t}+\beta_{3t}\theta_1 b_{2t-1}}\right)\left(I_{jt}^{\beta_{4t}+\beta_{3t}\theta_1 b_{4t-1}}\right)} \qquad (12)$$

Hence expression (12) is the simplified form of the model (3) after eliminating the variable: the stayers among past migrants; by expressing it in terms of current opportunities and intervening barriers. Comparing elasticities of $O_{jt}$, $O_{it}$, $I_{jt}$ in model (3) and (12) we can easily observe that the elasticities in model (3) give the volume and direction of the current effects of the variables whereas the elasticities in the model (12) give the cummulative effects of the volume and direction of the variables on the current migration.

Again as Singh and Yadava (1978-79) have said, here it is also possible to estimate the parameters involved in the model using a two stage procedure. If opportunities and intervening factors are known over a period of time, then the parameters $\gamma_i$, $\gamma_j$ and $h_j$ can be estimated and once $\gamma_i$ $\gamma_j$ $h_j$ and $\lambda$ are known, the remaining parameters can be easily estimated.

### 3. Application of the Model

Earlier Singh and Yadava (1974) studied the pattern of Rural-urban migration using the data collected in the Demographic survey of Varanasi (Rural) 1969-70. In that study the determinants of the Rural-urban migration was studied by using the following three models

(i) $Ms = \Lambda X_j + BY_j$

(ii) $M_j = \Lambda X_j + Be^{-zj}$

(iii) $M_j = X_j^{\Lambda} \cdot Y_j^{B}$

where $M_j$ = No. of migrants to the $j$th place of destination from fixed origin i during the period 1960-69.

$X_j$ = No. of migrants to the $j$th place of destination from fixed origin before the year 1960. (past migranti).

$$Y_j = \frac{p_j}{d_j} \text{ where } p_j = \frac{O_j}{O} \text{ and } O = \sum_{i=1}^{n} O^*$$

$$\text{and } d_j - \frac{D_j}{\Sigma D_j}$$

$O_j$ = No. of opportunities at the place of destination

$D_j$ = distance between fixed origin $i$ to a destination $j$

Using the urban population of the destination $j$ (= $u_j$) as a proxy to the opportunities at $j$, they analysed the data using all the three models and concluded that past migration plays a significant role in the determination of current flow of migration.

But the framework of the above models leads to much complication in determining the significance or non-significance of the variables because $Y_j$ is defined as the ratio of relative opportunities to the relative distance which in turn makes it a problem iu deciding the separate effect of opportunities as well as distance. Further the use of urban population ($u_j$) as proxy for the variable : opportunities ($O_j$) stands at variance for the reason behind its use, because the simple correlation between current migration flows and urban population at destination $j$ ($u_j$) is observed to be −0.7800. Also, there is no evidence given in the paper to justify the relationship, even though it is negative. Hence, the use of $u_j$ does not seem to be appropriate for the problem on hand,

To overcome the above limitations in the study (Singh and Yadava 1974), we have studied Rural-urban migration pattern using their own data (Singh and Yadava, 1974) with the help of our model (3) modified to suit the availability of data i.e. assuming that current migration flow is directly proportional to the stayers among past migrants and inversely proportional to the exponential form of distance function (i.e. a proxy to the intervening barriers). Mathematically, the modified form of model (3) used for the application is as follows :

$$M_{jt} = K \frac{S_{jt}^{\beta_1}}{e^{\beta_2 D_j}} \xi \tag{3.1}$$

where

$M_{jt}$ = Migration from rural area of Varanasi Tahsil to urban destination places 7 during the period 1960-69.

$S_{jt}$ = Migrants from rural area of Varanasi Tahsil to urban destination place $j$ migrated before 1960 and still residing in $j$.

$D_j$ — Distance from rural area of Varanasi Tahsil to urban destination $j$

$\xi$ = random error, $K$ = proportionality constant.

Using the data given below in Table 1 the parameters in the model were estimated using log linear regression form of the model. The results are given below in Table 2.

TABLE 1

| Variables | Destination Places | | | | |
|---|---|---|---|---|---|
| | Varanasi | U.P.* | W. Bengal | Maharashtra | Others |
| $M_j$ | 150 | 106 | 109 | 147 | 72 |
| $+S_j$ | 78 | 40 | 69 | 107 | 27 |
| $D$ (km) | 18 | 255 | 700 | 1250 | 1100 |

[a]average distance of all places which are included in this group.
*excluding Varanasi
SOURCE OF DATA : Singh and Yadava (1974).
+Singh and Yadava (1974) denoted it as $X_j$.

Values of the parameters estimated using log linear form of the model (3.1) and data given in Table 1, are given below in Table 2.

TABLE 2

| Value of Parameter | S, E. | t |
|---|---|---|
| $\beta_1$ = 0.4984619 | 0.09525 | 5.2332 |
| $\beta_2$ = -0.0001758 | 0.0000996 | 1.7649* |
| Interecrt = $\beta_0$ = log $K$ = 2.82462 | | |
| $R^2$ = 0.930353. | | |

*not significant at 5% level.

From Table 2 it is found that all the parameters are with expected signs but the parameter of the variable $D_j$ is not significant at 5% level. The model explains 93 percentage of variation which is appreciably high. Hence it can be said that the current flow of migration is determined by the number of stayers among past migrants from fixed origin to different destinations.

## 4 Conclusion

In this study the extended model of migration of Singh and Yadava is reformulated and is applied to the data on migration collected in the Demographic survey of Varanasi (Rural) 1960-69. The Model explains 93 percentage of variation. It is therefore concluded that current 0ow of migration is determined mainly by the stayers among the past migration between fixed origin to different destinations. However, an examination of the validity of the complete model (3) requires data on migration with fixed origin and different destinations along with sufficient information on the opportunities and intervening barriers.

## References


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